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Original Contribution

DIRECT BOUNDARY ELEMENT METHOD FOR COMPUTATION OF POTENTIAL FLOW AROUND A PROLATE SPHEROID

M. Mushtaq*[1], N. A. Shah[1], G. Muhammad[1]

Department of Mathematics, University of Engineering & Technology, Lahore – Pakistan

ABSTRACT
In this paper, a direct boundary element method is applied for computing potential flow around a Prolate Spheroid using quadrilateral elements. The computed results for flow velocities are compared with analytical results. The accuracy of the computed results is quite good.

Key words: Boundary element method, Potential flow, Axisymmetric flow, Prolate spheroid.

INTRODUCTION

Over the last twenty years, the term boundary element method has become more popular. The boundary element method is a numerical technique and hence it is an important subject of research amongst the numerically community. This method is derived through the discretisation of an integral equation that is mathematically equivalent to the original partial differential equation. It is easier to use and more efficient, cost effective and time saving, than the other competing computational methods i.e. finite difference and finite element methods. The boundary element method has a wide range of applications in fluid flow problems.

FLOW PAST A PROLATE SPHEROID

Let a Prolate spheroid be generated by rotating an ellipse of semi – major axis a and semi – minor axis b about its major axis and let a uniform stream of velocity U be in the positive direction of z – axis as shown in figure 1 .

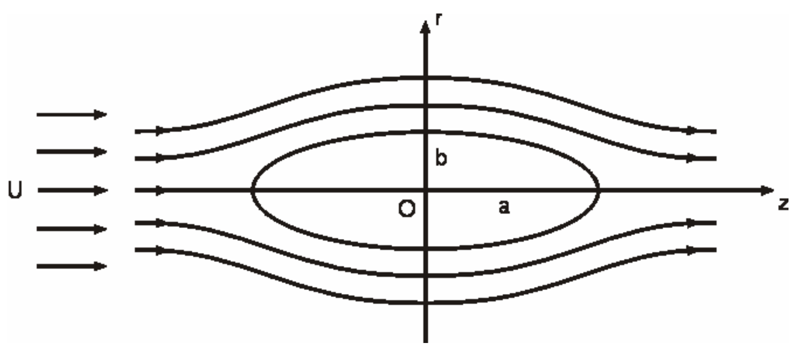


Fig. 1: The Flow Past a Prolate Spheroid

The Prolate spheroid is defined by the transformation

$$z + i r = c \cosh \zeta = c \cosh (\xi + i \eta)$$
$$= c \cosh \xi \cosh (i \eta) + c \sinh \xi \sinh (i \eta)$$
$$= c \cosh \xi \cos \eta + i c \sinh \xi \sin \eta$$

Comparison of real and imaginary parts gives

$$z = c \cosh \xi \cos \eta , \quad r = c \sinh \xi \sin \eta \quad (1)$$

Therefore the curve $\xi = \xi_0$ is an ellipse in the zr – plane whose semi – axes are

$$\left.\begin{aligned} a &= c \cosh \xi_0 \\ b &= c \sinh \xi_0 \end{aligned}\right] \quad (2)$$

and so $\xi = \xi_0$ is a Prolate spheroid .

* **Correspondence to:** Muhammad Mushtaq, Assistant Professor, Department of Mathematics University of Engineering & Technology, Lahore

The stream function ψ for a Prolate spheroid moving in the negative direction of the z – axis with velocity U is given by

$$\psi = \frac{\frac{1}{2} U b^2 \left(\cosh \xi + \sinh^2 \xi \ln \tanh \frac{\xi}{2} \right) \sin^2 \eta}{\frac{a}{c} + \frac{b^2}{c^2} \ln \frac{a+b-c}{a+b+c}} \quad (3)$$

Also , the stream function ψ for the uniform stream with velocity U , in the positive direction of z – axis is given by

$$\psi = -\frac{1}{2} U r^2$$

Therefore the stream function ψ for the streaming motion past a fixed Prolate spheroid in the positive direction of the z – axis becomes

$$\psi = -\frac{1}{2} U r^2 + \frac{\frac{1}{2} U b^2 \left(\cosh \xi + \sinh^2 \xi \ln \tanh \frac{\xi}{2} \right) \sin^2 \eta}{\frac{a}{c} + \frac{b^2}{c^2} \ln \frac{a+b-c}{a+b+c}} \quad (4)$$

which on using (1) becomes

$$\psi = -\frac{1}{2} U c^2 \sinh^2 \xi \sin^2 \eta + \frac{\frac{1}{2} U b^2 \left(\cosh \xi + \sinh^2 \xi \ln \tanh \frac{\xi}{2} \right) \sin^2 \eta}{\frac{a}{c} + \frac{b^2}{c^2} \ln \frac{a+b-c}{a+b+c}} \quad (5)$$

To determine the formula for the velocity , the following relation is used

$$V^2 r^2 f'(\zeta) \bar{f}'(\bar{\zeta}) = \left(\frac{\partial \psi}{\partial \xi} \right)^2 + \left(\frac{\partial \psi}{\partial \eta} \right)^2 \quad (6)$$

Since $f(\zeta) = c \cosh(\zeta)$

$f'(\zeta) = c \sinh(\zeta) = c \sinh(\xi + i\eta)$,

$\bar{f}'(\bar{\zeta}) = c \sinh(\xi - i\eta)$

and $f'(\zeta) \bar{f}'(\bar{\zeta}) =$

$$c^2 (\sinh^2 \xi \cos^2 \eta + \cosh^2 \xi \sin^2 \eta) \quad (7)$$

When $\xi = \xi_0$, then from (1) , (6) and (7)

$$V^2 c^4 \sinh^2 \xi_0 \sin^2 \eta \, (\sinh^2 \xi_0 \cos^2 \eta + \cosh^2 \xi_0 \sin^2 \eta) = \left(\frac{\partial \psi}{\partial \xi} \right)^2_{\xi=\xi_0} + \left(\frac{\partial \psi}{\partial \eta} \right)^2_{\xi=\xi_0} \quad (8)$$

Now from (5) , we get

$$= -U c^2 \sinh \xi_0 \cosh \xi_0 \sin^2 \eta + \frac{U b^2 \left(\sinh \xi_0 + \sinh \xi_0 \cosh \xi_0 \ln \tanh \frac{\xi_0}{2} \right) \sin^2 \eta}{\frac{a}{c} + \frac{b^2}{c^2} \ln \frac{a+b-c}{a+b+c}} \quad (9)$$

Since for a Prolate spheroid

$$a = c \cosh \xi_0 , \quad b = c \sinh \xi_0 \quad (10)$$

$$\text{But } \tanh \frac{\xi_0}{2} = \frac{a+b-c}{a+b+c} = \frac{b}{a+c} \quad (11)$$

From (9) , (10) , and (11) , we get

$$\left(\frac{\partial \psi}{\partial \xi} \right)_{\xi=\xi_0} = U \sin^2 \eta \left[-ab + \frac{\frac{b^3}{c} + \frac{ab^3}{c^2} \ln \frac{b}{a+c}}{\frac{a}{c} + \frac{b^2}{c^2} \ln \frac{b}{a+c}} \right] = U \sin^2 \eta \left[\frac{-cb}{\frac{a}{c} + \frac{b^2}{c^2} \ln \frac{b}{a+c}} \right] \quad (12)$$

and from (5) , (10) , and (11) , we obtain

$$\left(\frac{\partial \psi}{\partial \eta} \right)_{\xi=\xi_0} = 0 \quad (13)$$

Using (12) and (13) , (8) becomes

$$V^2 c^4 \sinh^2 \xi_0 \sin^2 \eta \, [\sinh^2 \xi_0 \cos^2 \eta + \cosh^2 \xi_0 \sin^2 \eta] = \frac{U^2 b^2 c^2 \sin^4 \eta}{\left[\frac{a}{c} + \frac{b^2}{c^2} \ln \frac{b}{a+c} \right]^2} \quad (14)$$

But from (1) and (2) , we get

$$\frac{z}{a} = \cos \eta , \quad \frac{r}{b} = \sin \eta \quad (15)$$

Using (10) , (15) in (14) , we have

$$V^2 = \frac{U^2 r^2 a^2 c^2}{\left[\frac{a}{c} + \frac{b^2}{c^2} \ln \frac{b}{a+c} \right]^2 (b^4 z^2 + a^4 r^2)} \quad (16)$$

Taking square root of (16) , the magnitude of exact velocity distribution over the boundary of a Prolate spheroid is given by

$$V = \frac{U a c r}{\left[\frac{a}{c} + \frac{b^2}{c^2} \ln \frac{b}{a+c} \right] \sqrt{b^4 z^2 + a^4 r^2}} \quad (17)$$

BOUNDARY CONDITIONS

The boundary condition to be satisfied over the surface of a Prolate spheroid is

$$\frac{\partial \phi_{p.s}}{\partial n} = U (\hat{n} \cdot \hat{k}) \quad (18)$$

where $\phi_{p.s}$ is the perturbation velocity potential of a Prolate spheroid and $\hat{n}$ is the outward drawn unit normal to the surface of a Prolate spheroid

The equation of the boundary of the Prolate spheroid

$$\frac{z^2}{a^2} + \frac{y^2}{b^2} + \frac{x^2}{b^2} = 1$$

Let $f(x, y, z) = \frac{z^2}{a^2} + \frac{y^2}{b^2} + \frac{x^2}{b^2} - 1$

Then $\nabla f = \frac{2x}{b^2}\hat{i} + \frac{2y}{b^2}\hat{j} + \frac{2z}{a^2}\hat{k}$

Therefore

$$\hat{n} = \frac{\nabla f}{|\nabla f|} = \frac{\frac{2x}{b^2}\hat{i} + \frac{2y}{b^2}\hat{j} + \frac{2z}{a^2}\hat{k}}{\sqrt{\left(\frac{2z}{a^2}\right)^2 + \left(\frac{2y}{b^2}\right)^2 + \left(\frac{2x}{b^2}\right)^2}}$$

Thus $$\hat{n} \cdot \hat{k} = \frac{\frac{2z}{a^2}}{\sqrt{\left(\frac{2z}{a^2}\right)^2 + \left(\frac{2y}{b^2}\right)^2 + \left(\frac{2x}{b^2}\right)^2}}$$

Therefore, the boundary condition in (18) takes the form

$$\frac{\partial \phi_{p.s}}{\partial n} = U \frac{\frac{z}{a^2}}{\frac{\sqrt{b^4 z^2 + a^4 y^2 + a^4 x^2}}{b^2 a^2}}$$

$$= \frac{z\, b^2}{\sqrt{b^4 z^2 + a^4 (y^2 + x^2)}}$$

(Taking U = 1) (19)

Equation (19) is the boundary condition which must be satisfied over the boundary of a Prolate spheroid.

DISCRETIZATION OF ELEMENTS

The direct boundary element method is applied to calculate the potential flow solution around the Prolate spheroid for which the analytical solution is available.

Consider the surface of the sphere in one octant to be divided into three quadrilateral elements by joining the centroid of the surface with the mid points of the curves in the coordinate planes as shown in figure 2 .

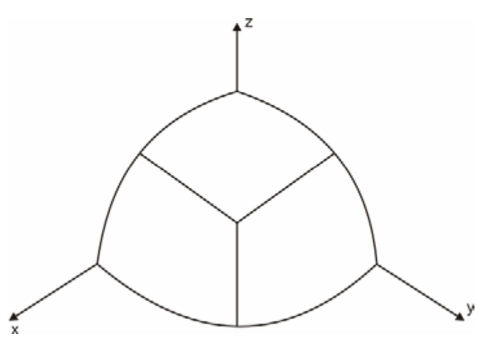


Fig. 2: Surface of the sphere divided into three quadrilateral elements

Then each element is divided further into four elements by joining the centroid of that element with the mid–point of each side of the element. Thus one octant of the surface of the sphere is divided into 12 elements and the whole surface of the body is divided into 96 boundary elements. The above mentioned method is adopted in order to produce a uniform distribution of element over the surface of the body.

Figure 3 shows the method for finding the coordinate (x_p, y_p, z_p) of any point P on the surface of the sphere.

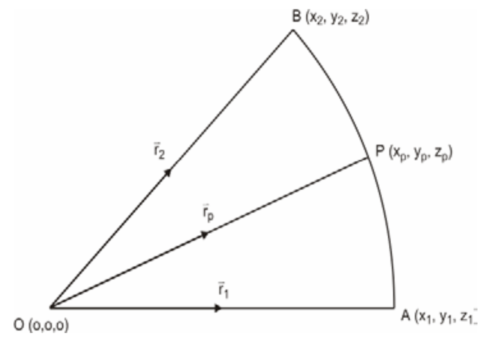


Fig. 3: The method of finding the coordinate (x_p,y_p,z_p) of any point P on the surface of the sphere

From figures 3 we have the following equation

$$|\vec{r}_p| = 1$$

$$\vec{r}_p \cdot \vec{r}_1 = \vec{r}_p \cdot \vec{r}_2$$

$$(\vec{r}_1 \times \vec{r}_2) \cdot \vec{r}_p = 0$$

or in cartesian form

$$x_p^2 + y_p^2 + z_p^2 = 1$$

$$x_p (x_1 - x_2) + y_p (y_1 - y_2) + z_p (z_1 - z_2) = 0$$

$$x_p (y_1 z_2 - z_1 y_2) + y_p (x_2 z_1 - x_1 z_2) + z_p (x_1 y_2 - x_2 y_1) = 0$$

As the body possesses planes of symmetry, this fact may be used in the input to the program and only the non–redundant portion need be specified by input points. The other portions are automatically taken into account. The planes of symmetry are taken to be the coordinate planes of the reference coordinate system. The advantage of the use of symmetry is that it reduces the order of the resulting system of equations and consequently reduces the computing time in running a program. As a sphere is symmetric with respect to all three coordinate planes of the reference coordinate system, only one eighth of the body surface need be specified by the input points, while the other seven–eighth can be accounted for by symmetry.

The Prolate spheroids of fineness ratios 2 and 10 are discretised into 24and 96 boundary elements and the computed velocity distributions are compared with analytical solutions for the Prolate spheroids. In both cases of spheroids, the input points are distributed on the surface of a sphere and the x and y-coordinates of these points are then divided by the fineness ratios to generate the points for the Prolate spheroids. The number of boundary elements used to obtain the computed velocity distribution are the same as are used for the sphere.

The calculated velocity distributions are compared with analytical solutions for the Prolate spheroid of fineness ratios 2 and 10 using Fortran programming.

Table 1: Comparison of the computed velocities with exact velocity over the surface of a Prolate spheroid with fineness ratio 2 using 24 boundary elements.

ELEMENT	XM	YM	ZM	R = $\sqrt{(YM)^2 + (ZM)^2}$	COMPUTED VELOCITY	EXACT VELOCITY
1	-.321E+00	-.374E+00	.161E+00	.40696E+00	.11589E+01	.11871E+01
2	-.748E+00	-.161E+00	.161E+00	.22706E+00	.76132E+00	.93409E+00
3	-.748E+00	.161E+00	.161E+00	.22706E+00	.76132E+00	.93409E+00
4	-.321E+00	.374E+00	.161E+00	.40696E+00	.11589E+01	.11871E+01
5	.321E+00	.374E+00	.161E+00	.40696E+00	.11589E+01	.11871E+01
6	.748E+00	.161E+00	.161E+00	.22706E+00	.76132E+00	.93409E+00
7	.748E+00	-.161E+00	.161E+00	.22706E+00	.76132E+00	.93409E+00
8	.321E+00	-.374E+00	.161E+00	.40696E+00	.11589E+01	.11871E+01
9	-.321E+00	-.161E+00	.374E+00	.40696E+00	.11589E+01	.11871E+01
10	-.321E+00	.161E+00	.374E+00	.40696E+00	.11589E+01	.11871E+01
11	.321E+00	.161E+00	.374E+00	.40696E+00	.11589E+01	.11871E+01
12	.321E+00	-.161E+00	.374E+00	.40696E+00	.11589E+01	.11871E+01

Graph. 1 Comparison of computed and analytical velocity distributions over the surface of a Prolate spheroid using 24 boundary elements with fineness ratio 2

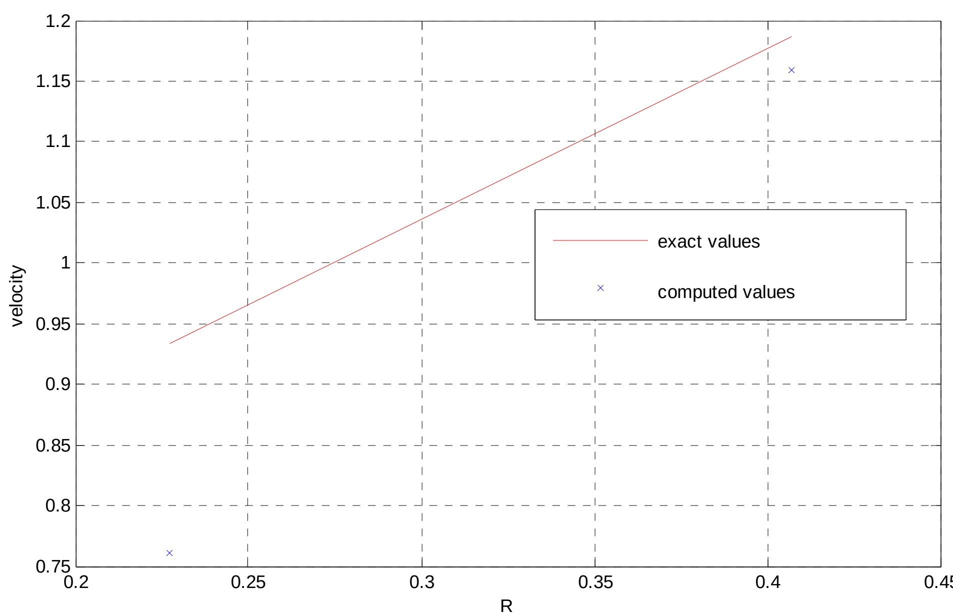


Table 2: Comparison of the computed velocities with exact velocity over the surface of a Prolate spheroid with fineness ratio 10 using 24 elements.

ELEMENT	XM	YM	ZM	R = $\sqrt{(YM)^2+(ZM)^2}$	COMPUTED VELOCITY	EXACT VELOCITY
1	-.321E+00	-.748E-01	.321E-01	.81391E-01	.10311E+01	.10199E+01
2	-.748E+00	-.321E-01	.321E-01	.45412E-01	.98097E+00	.10071E+01
3	-.748E+00	.321E-01	.321E-01	.45412E-01	.98097E+00	.10071E+01
4	-.321E+00	.748E-01	.321E-01	.81391E-01	.10311E+01	.10199E+01
5	.321E+00	.748E-01	.321E-01	.81391E-01	.10311E+01	.10199E+01
6	.748E+00	.321E-01	.321E-01	.45412E-01	.98097E+00	.10071E+01
7	.748E+00	-.321E-01	.321E-01	.45412E-01	.98097E+00	.10071E+01
8	.321E+00	-.748E-01	.321E-01	.81391E-01	.10311E+01	.10199E+01
9	-.321E+00	-.321E-01	.748E-01	.81391E-01	.10311E+01	.10199E+01
10	-.321E+00	.321E-01	.748E-01	.81391E-01	.10311E+01	.10199E+01
11	.321E+00	.321E-01	.748E-01	.81391E-01	.10311E+01	.10199E+01
12	.321E+00	-.321E-01	.748E-01	.81391E-01	.10311E+01	.10199E+01

Graph. 2: Comparison of computed and analytical velocity distributions over the surface of a Prolate spheroid using 24 boundary elements with fineness ratio 10

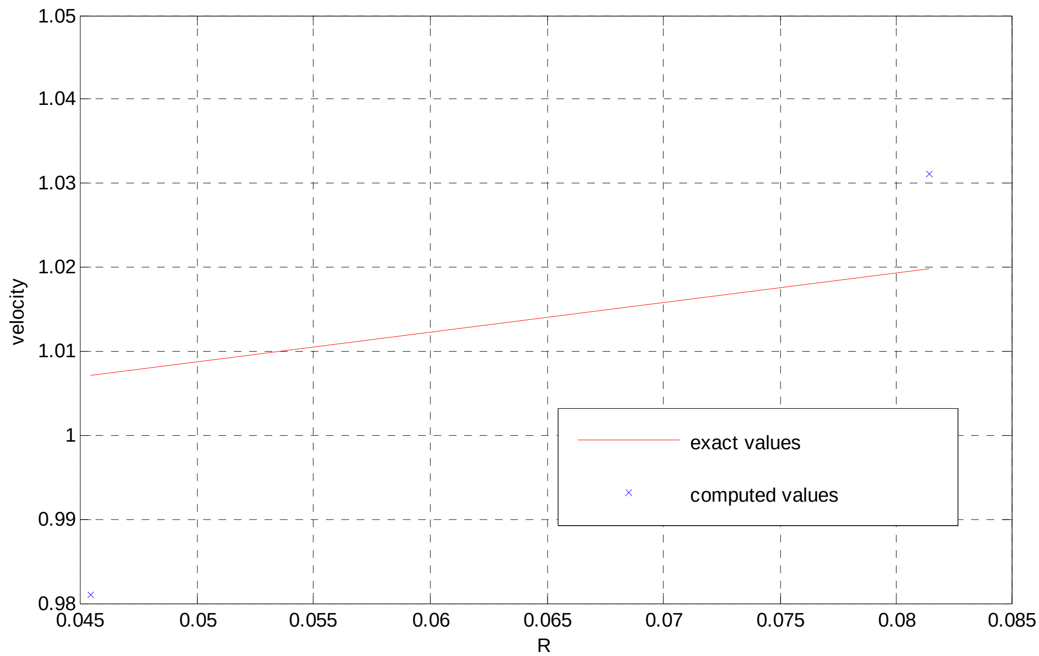


Table 3: Comparison of the computed velocities with exact velocity over the surface of a Prolate spheroid with fineness ratio 2 using 96 boundary elements.

ELEMENT	XM	YM	ZM	R = $\sqrt{(YM)^2+(ZM)^2}$	COMPUTED VELOCITY	EXACT VELOCITY
1	-.177E+00	-.467E+00	.885E-01	.47529E+00	.11975E+01	.12048E+01
2	-.522E+00	-.399E+00	.785E-01	.40676E+00	.11385E+01	.11521E+01
3	-.798E+00	-.261E+00	.785E-01	.27264E+00	.93605E+00	.97640E+00
4	-.934E+00	-.885E-01	.885E-01	.12511E+00	.47752E+00	.57150E+00
5	-.934E+00	.885E-01	.885E-01	.12511E+00	.47752E+00	.57150E+00
6	-.798E+00	.261E+00	.785E-01	.27264E+00	.93605E+00	.97640E+00
7	-.522E+00	.399E+00	.785E-01	.40676E+00	.11385E+01	.11521E+01
8	-.177E+00	.467E+00	.885E-01	.47529E+00	.11975E+01	.12048E+01
9	.177E+00	.467E+00	.885E-01	.47529E+00	.11975E+01	.12048E+01
10	.522E+00	.399E+00	.785E-01	.40676E+00	.11385E+01	.11521E+01

11	.798E+00	.261E+00	.785E-01	.27264E+00	.93605E+00	.97640E+00
12	.934E+00	.885E-01	.885E-01	.12511E+00	.47752E+00	.57150E+00
13	.934E+00	-.885E-01	.885E-01	.12511E+00	.47752E+00	.57150E+00
14	.798E+00	-.261E+00	.785E-01	.27264E+00	.93605E+00	.97640E+00
15	.522E+00	-.399E+00	.785E-01	.40676E+00	.11385E+01	.11521E+01
16	.177E+00	-.467E+00	.885E-01	.47529E+00	.11975E+01	.12048E+01
17	-.157E+00	-.399E+00	.261E+00	.47693E+00	.11977E+01	.12059E+01
18	-.470E+00	-.352E+00	.235E+00	.42289E+00	.11622E+01	.11659E+01
19	-.703E+00	-.235E+00	.235E+00	.33220E+00	.10100E+01	.10695E+01
20	-.798E+00	-.785E-01	.261E+00	.27264E+00	.93605E+00	.97640E+00
21	-.798E+00	.785E-01	.261E+00	.27264E+00	.93605E+00	.97640E+00
22	-.703E+00	.235E+00	.235E+00	.33220E+00	.10100E+01	.10695E+01
23	-.470E+00	.352E+00	.235E+00	.42289E+00	.11622E+01	.11659E+01
24	-.157E+00	.399E+00	.261E+00	.47693E+00	.11977E+01	.12059E+01
25	.157E+00	.399E+00	.261E+00	.47693E+00	.11977E+01	.12059E+01
26	.470E+00	.352E+00	.235E+00	.42289E+00	.11622E+01	.11659E+01
27	.703E+00	.235E+00	.235E+00	.33220E+00	.10100E+01	.10695E+01
28	.798E+00	.785E-01	.261E+00	.27264E+00	.93605E+00	.97640E+00
29	.798E+00	-.785E-01	.261E+00	.27264E+00	.93605E+00	.97640E+00
30	.703E+00	-.235E+00	.235E+00	.33220E+00	.10100E+01	.10695E+01
31	.470E+00	-.352E+00	.235E+00	.42289E+00	.11622E+01	.11659E+01
32	.157E+00	-.399E+00	.261E+00	.47693E+00	.11977E+01	.12059E+01
33	-.157E+00	-.261E+00	.399E+00	.47693E+00	.11977E+01	.12059E+01
34	-.470E+00	-.235E+00	.352E+00	.42289E+00	.11622E+01	.11659E+01
35	-.522E+00	-.785E-01	.399E+00	.40676E+00	.11385E+01	.11521E+01
36	-.522E+00	.785E-01	.399E+00	.40676E+00	.11385E+01	.11521E+01
37	-.470E+00	.235E+00	.352E+00	.42289E+00	.11622E+01	.11659E+01
38	-.157E+00	.261E+00	.399E+00	.47693E+00	.11977E+01	.12059E+01
39	.157E+00	.261E+00	.399E+00	.47693E+00	.11977E+01	.12059E+01
40	.470E+00	.235E+00	.352E+00	.42289E+00	.11622E+01	.11659E+01
41	.522E+00	.785E-01	.399E+00	.40676E+00	.11385E+01	.11521E+01
42	.522E+00	-.785E-01	.399E+00	.40676E+00	.11385E+01	.11521E+01
43	.470E+00	-.235E+00	.352E+00	.42289E+00	.11622E+01	.11659E+01
44	.157E+00	-.261E+00	.399E+00	.47693E+00	.11977E+01	.12059E+01
45	-.177E+00	-.885E-01	.467E+00	.47529E+00	.11975E+01	.12048E+01
46	-.177E+00	.885E-01	.467E+00	.47529E+00	.11975E+01	.12048E+01
47	.177E+00	.885E-01	.467E+00	.47529E+00	.11975E+01	.12048E+01
48	.177E+00	-.885E-01	.467E+00	.47529E+00	.11975E+01	.12048E+01

Graph. 3: Comparison of computed and analytical velocity distributions over the surface of a Prolate spheroid using 96 boundary elements with fineness ratio 2

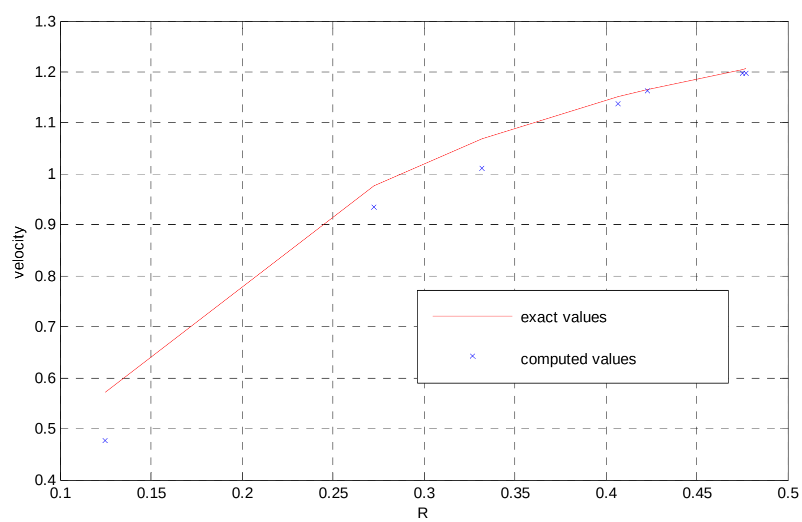
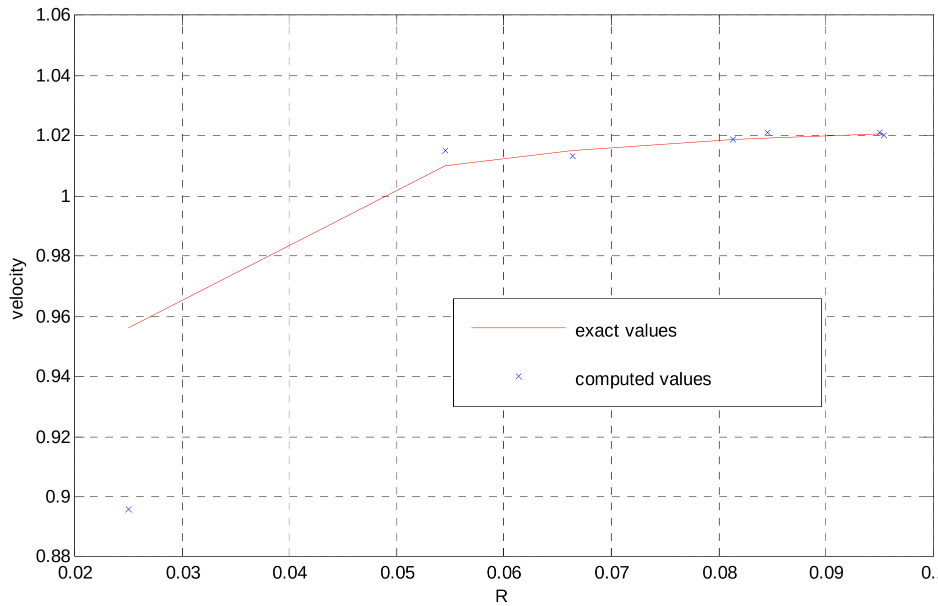


Table 4: Comparison of the computed velocities with exact velocity over the surface of a Prolate spheroid with fineness ratio 10 using 96 boundary elements.

ELEMENT	XM	YM	ZM	R = $\sqrt{(YM)^2 + (ZM)^2}$	COMPUTED VELOCITY	EXACT VELOCITY
1	-.177E+00	-.934E-01	.177E-01	.95057E-01	.10210E+01	.10205E+01
2	-.522E+00	-.798E-01	.157E-01	.81353E-01	.10186E+01	.10186E+01
3	-.798E+00	-.522E-01	.157E-01	.54527E-01	.10151E+01	.10099E+01
4	-.934E+00	-.177E-01	.177E-01	.25022E-01	.89600E+00	.95627E+00
5	-.934E+00	.177E-01	.177E-01	.25022E-01	.89600E+00	.95627E+00
6	-.798E+00	.522E-01	.157E-01	.54527E-01	.10151E+01	.10099E+01
7	-.522E+00	.798E-01	.157E-01	.81353E-01	.10186E+01	.10186E+01
8	-.177E+00	.934E-01	.177E-01	.95057E-01	.10210E+01	.10205E+01
9	.177E+00	.934E-01	.177E-01	.95057E-01	.10210E+01	.10205E+01
10	.522E+00	.798E-01	.157E-01	.81353E-01	.10186E+01	.10186E+01
11	.798E+00	.522E-01	.157E-01	.54527E-01	.10151E+01	.10099E+01
12	.934E+00	.177E-01	.177E-01	.25022E-01	.89600E+00	.95627E+00
13	.934E+00	-.177E-01	.177E-01	.25022E-01	.89600E+00	.95627E+00
14	.798E+00	-.522E-01	.157E-01	.54527E-01	.10151E+01	.10099E+01
15	.522E+00	-.798E-01	.157E-01	.81353E-01	.10186E+01	.10186E+01
16	.177E+00	-.934E-01	.177E-01	.95057E-01	.10210E+01	.10205E+01
17	-.157E+00	-.798E-01	.522E-01	.95386E-01	.10203E+01	.10206E+01
18	-.470E+00	-.703E-01	.470E-01	.84578E-01	.10211E+01	.10191E+01
19	-.703E+00	-.470E-01	.470E-01	.66440E-01	.10135E+01	.10150E+01
20	-.798E+00	-.157E-01	.522E-01	.54527E-01	.10151E+01	.10099E+01
21	-.798E+00	.157E-01	.522E-01	.54527E-01	.10151E+01	.10099E+01
22	-.703E+00	.470E-01	.470E-01	.66440E-01	.10135E+01	.10150E+01
23	-.470E+00	.703E-01	.470E-01	.84578E-01	.10211E+01	.10191E+01
24	-.157E+00	.798E-01	.522E-01	.95386E-01	.10203E+01	.10206E+01
25	.157E+00	.798E-01	.522E-01	.95386E-01	.10203E+01	.10206E+01
26	.470E+00	.703E-01	.470E-01	.84578E-01	.10211E+01	.10191E+01
27	.703E+00	.470E-01	.470E-01	.66440E-01	.10135E+01	.10150E+01
28	.798E+00	.157E-01	.522E-01	.54527E-01	.10151E+01	.10099E+01
29	.798E+00	-.157E-01	.522E-01	.54527E-01	.10151E+01	.10099E+01
30	.703E+00	-.470E-01	.470E-01	.66440E-01	.10135E+01	.10150E+01
31	.470E+00	-.703E-01	.470E-01	.84578E-01	.10211E+01	.10191E+01
32	.157E+00	-.798E-01	.522E-01	.95386E-01	.10203E+01	.10206E+01
33	-.157E+00	-.522E-01	.798E-01	.95386E-01	.10203E+01	.10206E+01
34	-.470E+00	-.470E-01	.703E-01	.84578E-01	.10211E+01	.10191E+01
35	-.522E+00	-.157E-01	.798E-01	.81353E-01	.10186E+01	.10186E+01
36	-.522E+00	.157E-01	.798E-01	.81353E-01	.10186E+01	.10186E+01
37	-.470E+00	.470E-01	.703E-01	.84578E-01	.10211E+01	.10191E+01
38	-.157E+00	.522E-01	.798E-01	.95386E-01	.10203E+01	.10206E+01
39	.157E+00	.522E-01	.798E-01	.95386E-01	.10203E+01	.10206E+01
40	.470E+00	.470E-01	.703E-01	.84578E-01	.10211E+01	.10191E+01
41	.522E+00	.157E-01	.798E-01	.81353E-01	.10186E+01	.10186E+01
42	.522E+00	-.157E-01	.798E-01	.81353E-01	.10186E+01	.10186E+01
43	.470E+00	-.470E-01	.703E-01	.84578E-01	.10211E+01	.10191E+01
44	.157E+00	-.522E-01	.798E-01	.95386E-01	.10203E+01	.10206E+01
45	-.177E+00	-.177E-01	.934E-01	.95057E-01	.10210E+01	.10205E+01
46	-.177E+00	.177E-01	.934E-01	.95057E-01	.10210E+01	.10205E+01
47	.177E+00	.177E-01	.934E-01	.95057E-01	.10210E+01	.10205E+01
48	.177E+00	-.177E-01	.934E-01	.95057E-01	.10210E+01	.10205E+01

Graph. 4: Comparison of computed and analytical velocity distributions over the Surface of a Prolate spheroid using 96 boundary elements with fineness ratio 10



Graphs 1 and 3 show the comparison of the computed and analytical distributions over the surface of a Prolate spheroid of fineness ratio 2 for 24 and 96 boundary elements respectively. The graphs 2 and 4 show the comparison of the computed and analytical distributions over the surface of a Prolate spheroid of fineness ratio 10 for 24 and 96 boundary elements respectively. The accuracy increases with the increase of number of boundary elements and fineness ratio.

CONCLUSION

A direct boundary element method is applied for computing potential flow around a Prolate spheroid. The computed flow velocities obtained by this method are compared with the analytical solutions for flow past a Prolate spheroid. It is found that the computed results for velocity distribution are good in agreement with the analytical results for the body under consideration.

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